LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – STATISTICS

FIFTH SEMESTER – NOVEMBER 2009

ST 5400 - APPLIED STOCHASTIC PROCESSES

Date & Time: 12/11/2009 / 9:00 - 12:00 Dept. No. Max. : 100 Marks

SECTION A

Answer all questions. **(10x2=20)**

1. Define Stochastic Process.
2. Give an example for a Markov Process.
3. Mention the conditions for a state of a Markov chain to be recurrent.
4. The probability of a particle reaching state A from state B is 0.7 and reaching state B from state A is 0.4. Find the periodicity of state A.
5. Define an Irreducible Markov chain
6. What is meant by one dimensional random walk?
7. When are two states said to be accessible from each other?
8. What information does an infinitesimal generator matrix provide?
9. Justify the following statement: "An absorbing state is always recurrent".
10. Define Martingale.

SECTION B

Answer any FIVE questions. **(5x8=40)**

11. A Markov chain X_0, X_1, X_2 has the transition probability matrix

$$P = \begin{array}{c|ccc} & 0 & 1 & 2 \\ \hline 0 & 0.3 & 0.2 & 0.5 \\ 1 & 0.5 & 0.1 & 0.4 \\ 2 & 0.5 & 0.2 & 0.3 \end{array}$$ and initial distribution $P[X_0 = j] = \frac{1}{3},\ j = 0,1,2$. Find

$P[X_0 = 2, X_1 = 1, X_2 = 0]$ and $P[X_0 = 1, X_1 = 1, X_2 = 1]$.

12. Explain the different types of stochastic processes.
13. Two vendors A and B, compete for the same market. Currently, customer choice of these vendors can be represented by the transition probability

matrix $$P = \begin{array}{c|cc} & A & B \\ \hline A & 0.5 & 0.5 \\ B & 0.6 & 0.4 \end{array}$$. What is A's long-term customer share of the market?

14. Show that the stationary distribution of a Markov chain with the transition probability matrix

$$P = \begin{array}{c|cccccc} & 0 & 1 & 2 & 3 & \cdots & \cdots \\ \hline 0 & q & p & 0 & 0 & \cdots & \cdots \\ 1 & q & 0 & p & 0 & \cdots & \cdots \\ 2 & q & 0 & 0 & p & 0 & \cdots \\ 3 & q & 0 & 0 & 0 & p & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{array}$$

such that $p+q=1$ is geometric distribution.

15. If $i \leftrightarrow j$ and i is recurrent, then show that j is also recurrent
16. Define Poisson process.
17. State and prove the additive property of the Poisson process
18. Give an example each for the following:
    (i) Poisson process (ii) Pure Birth process (iii) Discrete space – Discrete time process and (iv) Continuous space – Continuous time process.

SECTION C

Answer any TWO questions. (2x20=40)

19. a.) Find $\lim_{n\to\infty} p_{22}^{(n)}$ for a Markov chain with transition probability matrix

$$P = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 0.4 & 0.5 & 0.1 \\ 2 & 0.05 & 0.7 & 0.25 \\ 3 & 0.05 & 0.5 & 0.45 \end{array}.$$

    b.) Write short notes on the concepts of recurrence and transience. (12+8)
20. Derive the Kolmogorov forward and backward differential equations of a Markov process.
21. a.) Derive the general form of infinitesimal generator matrix of a Markov process.
    b.) Consider a Markov chain with state space $S=\{0,1\}$ with transition probability matrix

$$P = \begin{array}{c|cc} & 0 & 1 \\ \hline 0 & 1-\alpha & \alpha \\ 1 & \beta & 1-\beta \end{array}, \quad 0<\alpha,\beta<1.$$

    Calculate the mean recurrence time of state ‘0’ and verify that it is the reciprocal of Π_0, the stationary probability of state ‘0’. (10+10)
22. Derive the distribution $X(t)$, when $\{X(t), t\ge 0\}$ is a Poisson process.